

# LECTURES ABOUT (ADVANCED) STATISTICAL PHYSICS

*T.S.Biró, MTA Wigner Research Centre for Physics, Budapest*

*Lectures given at: University of Johannesburg, South-Africa,*

*November 26 – November 29, 2012.*

- 1. Ancient Thermodynamics (... - 1870)**
- 2. The Rise of Statistical Physics (1890 – 1920)**
- 3. Modern (postwar) Problems (1940 – 1980)**
- 4. Corrections (1950 – 2005)**
- 5. Generalizations (1960 – 2010)**
- 6. High Energy Physics (1950 – 2010)**

# LECTURE FIVE ABOUT (ADVANCED) STATISTICAL PHYSICS

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# FAQ

- **What is actually non-perturbative at  $T > T_c$  ? (low  $Q^2$  pairs)**
- **What is non-particle like (stringy) at  $T > T_c$  ? (interaction)**
- **Are all color charges stringed at  $T > T_c$ ? No! (a small fraction suffices for the effect.)**
- **Asymptotic freedom is incomplete at any finite temperature. Why and what is the quantitative measure of this effect? →**

# Arguments

## Non-perturbative effects at arbitrary high temperature:

1. Thermal distribution of  $Q^2$
2. NP order parameter: cut-off in  $Q^2$
3. Its thermal expectation value  $\rightarrow$  order of NP effects
4. high-T expansion
5.  $\rightarrow$  high-T NP terms in EoS (pressure, int.measure)

# Pressure: NP effects at any T

$$p = p_P \int_{\Lambda^2}^{\infty} P(Q^2) dQ^2 + p_{NP} \int_0^{\Lambda^2} P(Q^2) dQ^2$$

$$\int_0^{\Lambda^2} P(Q^2) dQ^2 = \int_0^{\Lambda^2/T^2} f(x) dx$$

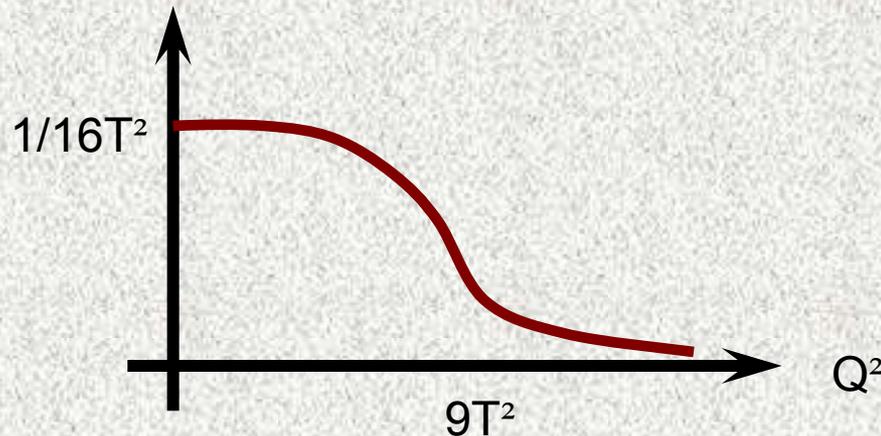
$$p = p_p - (p_p - p_{NP}) \begin{cases} 1 & (T \ll \Lambda) \\ \Lambda^2/T^2 f(0) & (T \gg \Lambda) \end{cases}$$

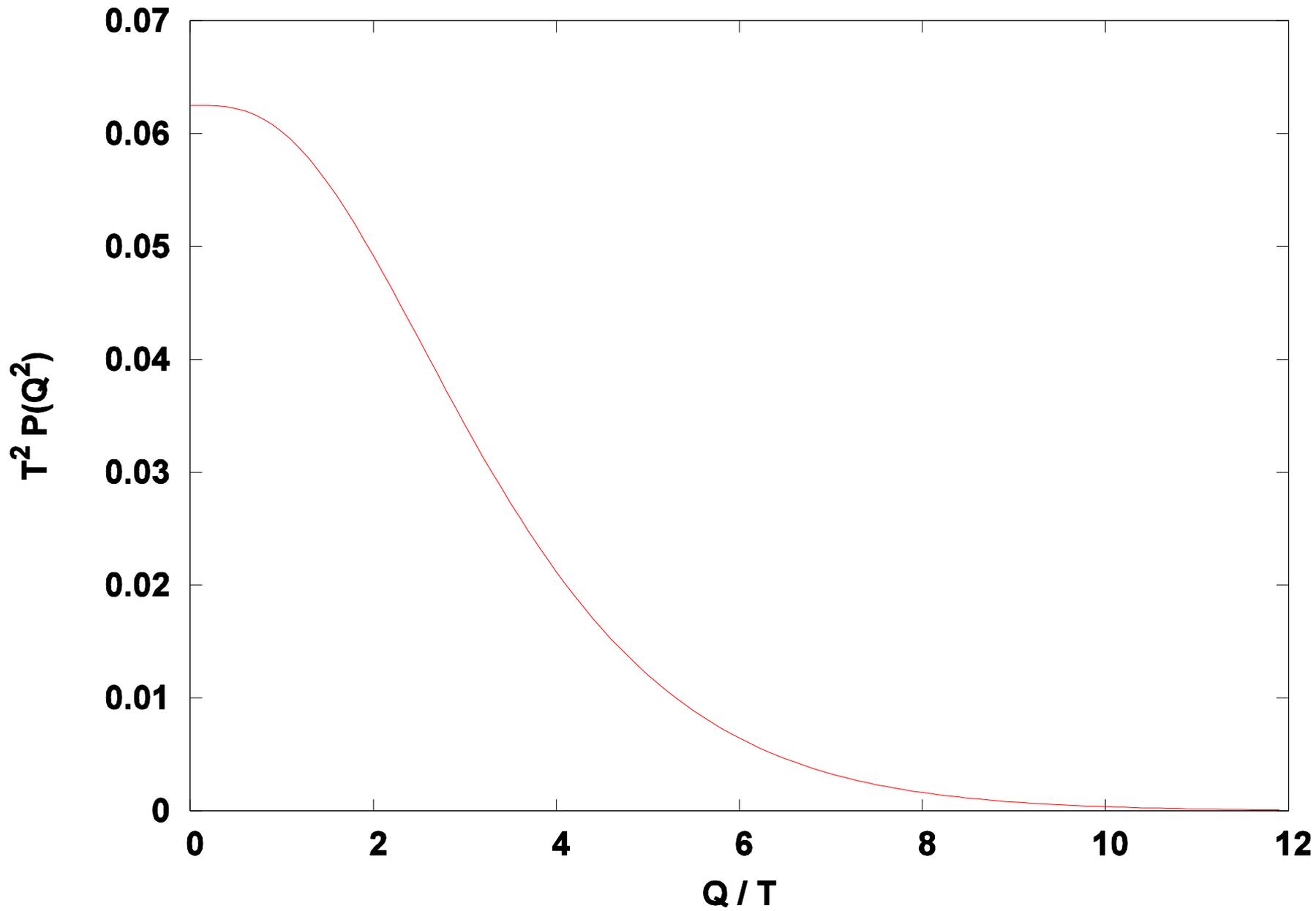
If it were  $f(0) = 0$ , then the QGP pressure would be free of NP effects!

# Thermal distribution of $Q^2$

$$P(Q^2) = \frac{\iint dE_1 dE_2 d\theta E_1^2 E_2^2 e^{-\beta(E_1+E_2)} \delta(Q^2 - 2E_1 E_2 (1 - \cos \theta))}{\iint dE_1 dE_2 d\theta E_1^2 E_2^2 e^{-\beta(E_1+E_2)}}$$

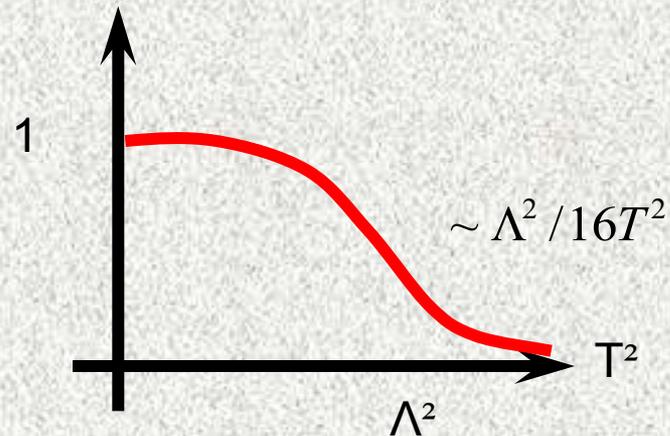
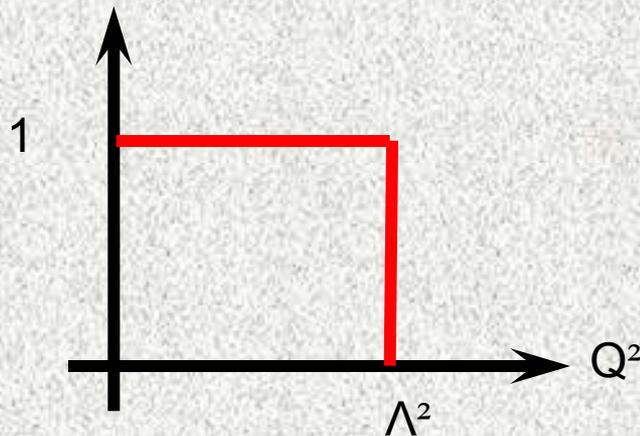
$$P(Q^2) = \frac{1}{64T^2} \left( \frac{Q^3}{T^3} K_1\left(\frac{Q}{T}\right) + 2 \frac{Q^2}{T^2} K_2\left(\frac{Q}{T}\right) \right)$$

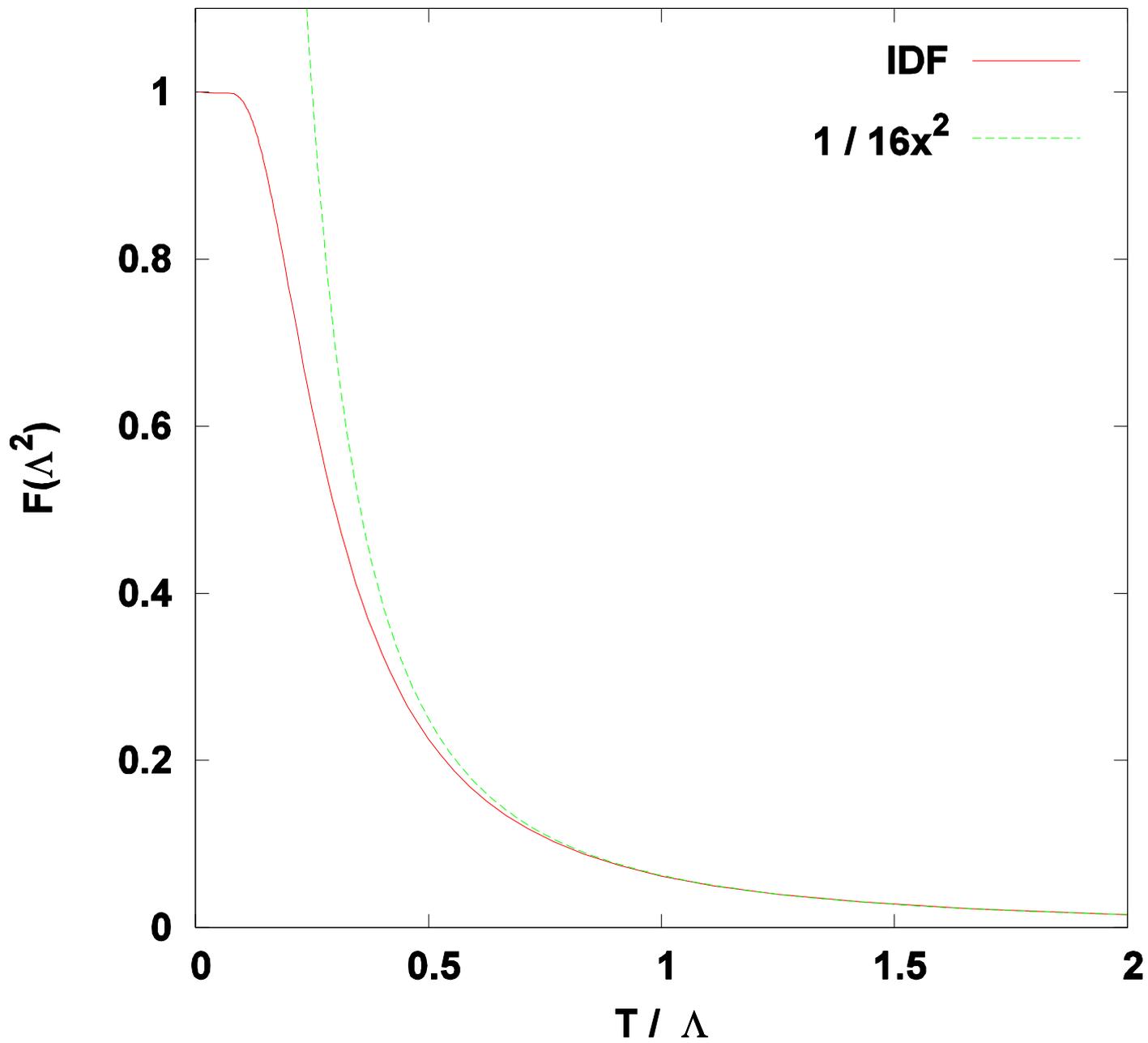




# Thermal expectation of NP order parameter

$$\langle \Theta(\Lambda^2 - Q^2) \rangle = \int_0^{\Lambda^2} P(Q^2) dQ^2$$

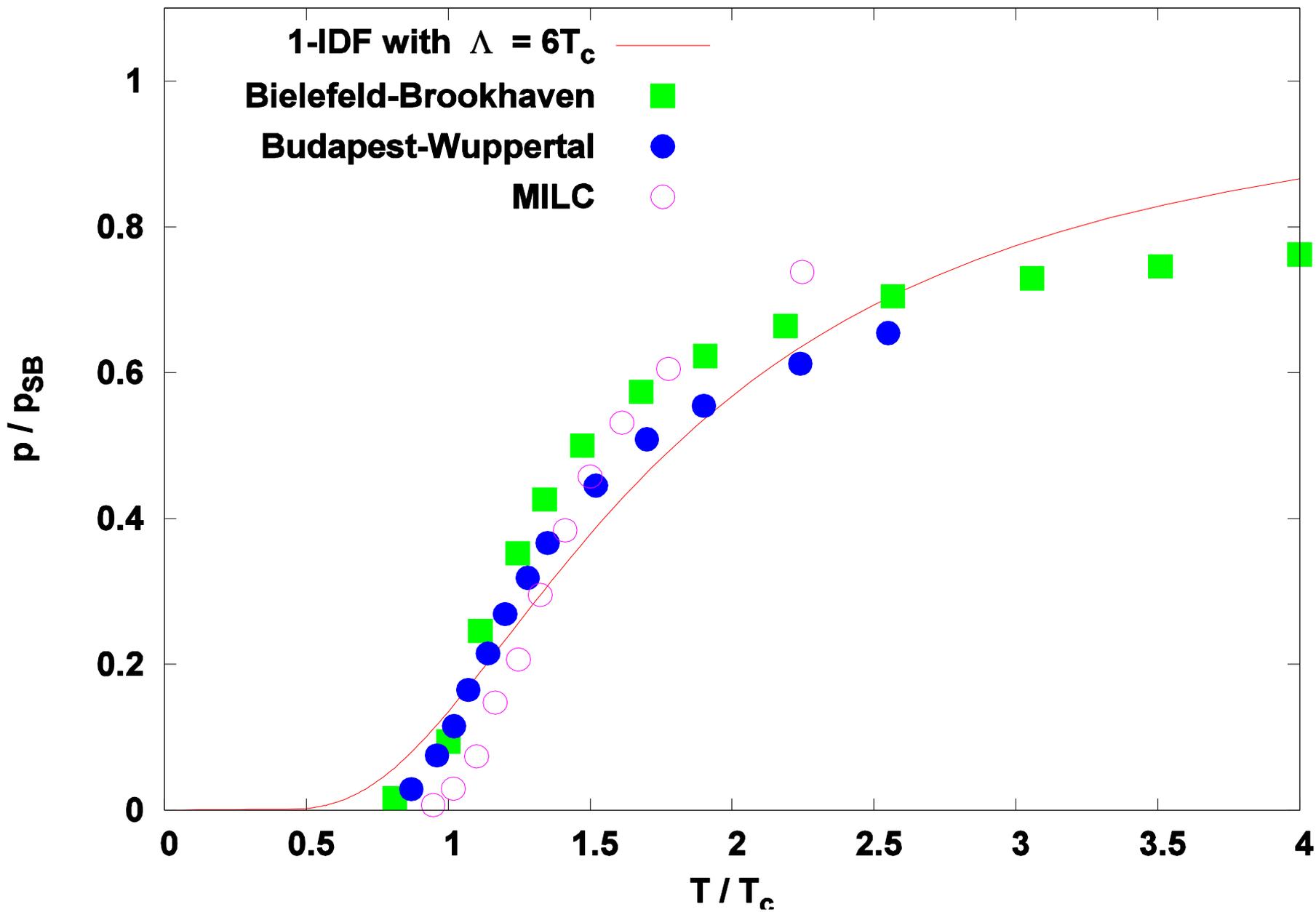


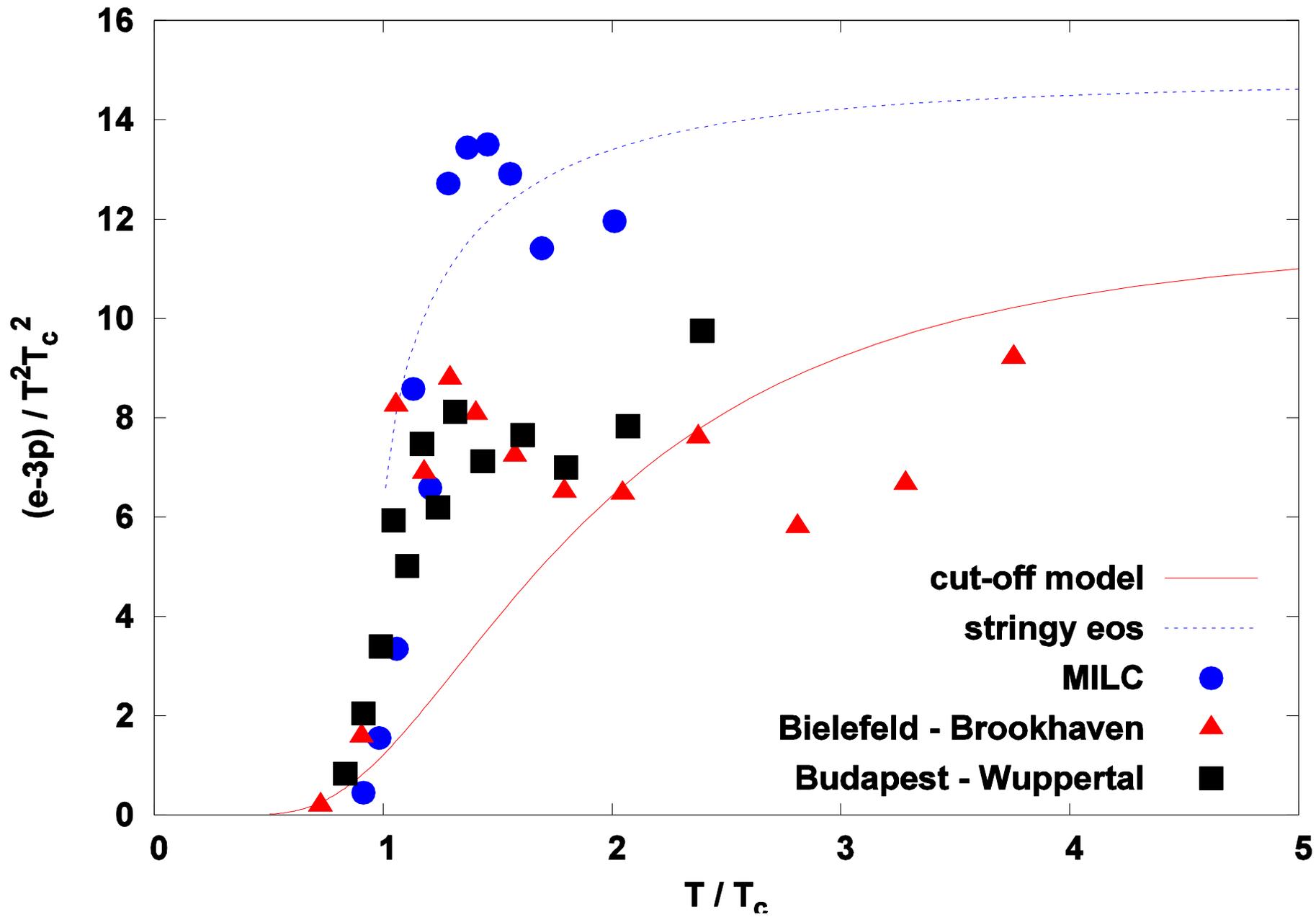


# NP effects at high T in the EoS

$$p = \frac{1}{3} \kappa T^4 - c_{NP} \Lambda^2 T^2$$

$$e - 3p = 2c_{NP} \Lambda^2 T^2$$







# WIKIPEDIA: TSALLIS ENTROPY

In physics, the **Tsallis entropy** is a generalization of the standard Boltzmann-Gibbs entropy. It was introduced in 1988 by Constantino Tsallis <sup>[1]</sup> as a basis for generalizing the standard statistical mechanics. In the scientific literature, the physical relevance of the Tsallis entropy was occasionally debated. However, from the years 2000 on, an increasingly wide spectrum of natural, artificial and social complex systems have been identified which confirm the predictions and consequences that are derived from this nonadditive entropy, such as nonextensive statistical mechanics <sup>[2]</sup>, which generalizes the Boltzmann-Gibbs theory.

Among the various experimental verifications and applications presently available in the literature, the following ones deserve a special mention:

The distribution characterizing the motion of cold atoms in dissipative optical lattices, predicted in 2003 <sup>[3]</sup> and observed in 2006 <sup>[4]</sup>.

The fluctuations of the magnetic field in the solar wind enabled the calculation of the q-triplet (or Tsallis triplet) <sup>[5]</sup>.

The velocity distributions in driven dissipative dusty plasma <sup>[6]</sup>. Spin glass relaxation <sup>[7]</sup>.

Trapped ion interacting with a classical buffer gas <sup>[8]</sup>.

High energy collisional experiments at LHC/CERN (CMS, ATLAS and ALICE detectors) <sup>[9]</sup> <sup>[10]</sup> and RHIC/Brookhaven (STAR and PHENIX detectors) <sup>[11]</sup>.

Among the various available theoretical results which clarify the physical conditions under which Tsallis entropy and associated statistics apply, the following ones can be selected:

Anomalous diffusion <sup>[12]</sup> <sup>[13]</sup>.

Uniqueness theorem <sup>[14]</sup>.

Sensitivity to initial conditions and entropy production at the edge of chaos <sup>[15]</sup> <sup>[16]</sup>.

Probability sets which make the nonadditive Tsallis entropy to be extensive in the thermodynamical sense <sup>[17]</sup>.

Strongly quantum entangled systems and thermodynamics <sup>[18]</sup>.

Thermostatistics of overdamped motion of interacting particles <sup>[19]</sup> <sup>[20]</sup>.

Nonlinear generalizations of the Schroedinger, Klein-Gordon and Dirac equations <sup>[21]</sup>.

For further details a bibliography is available at <http://tsallis.cat.cbpf.br/biblio.htm>

# Some applications

- Reservoir = QGP at constant volume
- Reservoir = QGP at constant pressure
- Reservoir = QGP at constant entropy
- Reservoir = classical Yang-Mills on lattice
- Reservoir = (Schwarzschild) black hole

# Heat capacity of QGP reservoir

- MIT bag model:

$$\triangleright E = V(\sigma T^4 + B), \quad p = \frac{\sigma T^4}{3} - B, \quad S = 4\sigma VT^3/3$$

$$C = \frac{dE}{dT} = 4\sigma VT^3 + (\sigma T^4 + B) \frac{dV}{dT}$$

# Heat capacity of QGP reservoir

- MIT bag model:

$$\blacktriangleright E = V(\sigma T^4 + B), \quad p = \frac{\sigma T^4}{3} - B, \quad S = 4\sigma VT^3/3$$

$$C_V = 4\sigma VT^3 = 3S, \quad C_p = \infty, \quad C_S = \frac{3}{4}S \left(1 - \frac{T^4}{T_0^4}\right)$$

$V$  const.  $T_{fit} = T \lim_{C \rightarrow \infty} e^{-S/C} = e^{-1/3} \approx 0.7$

# Heat capacity of QGP reservoir

- Chaotic classical Yang-Mills:
  - $S(E) = C_0 \ln(1 + E/C_0 T_0)$ , constant heat capacity  $C$  !

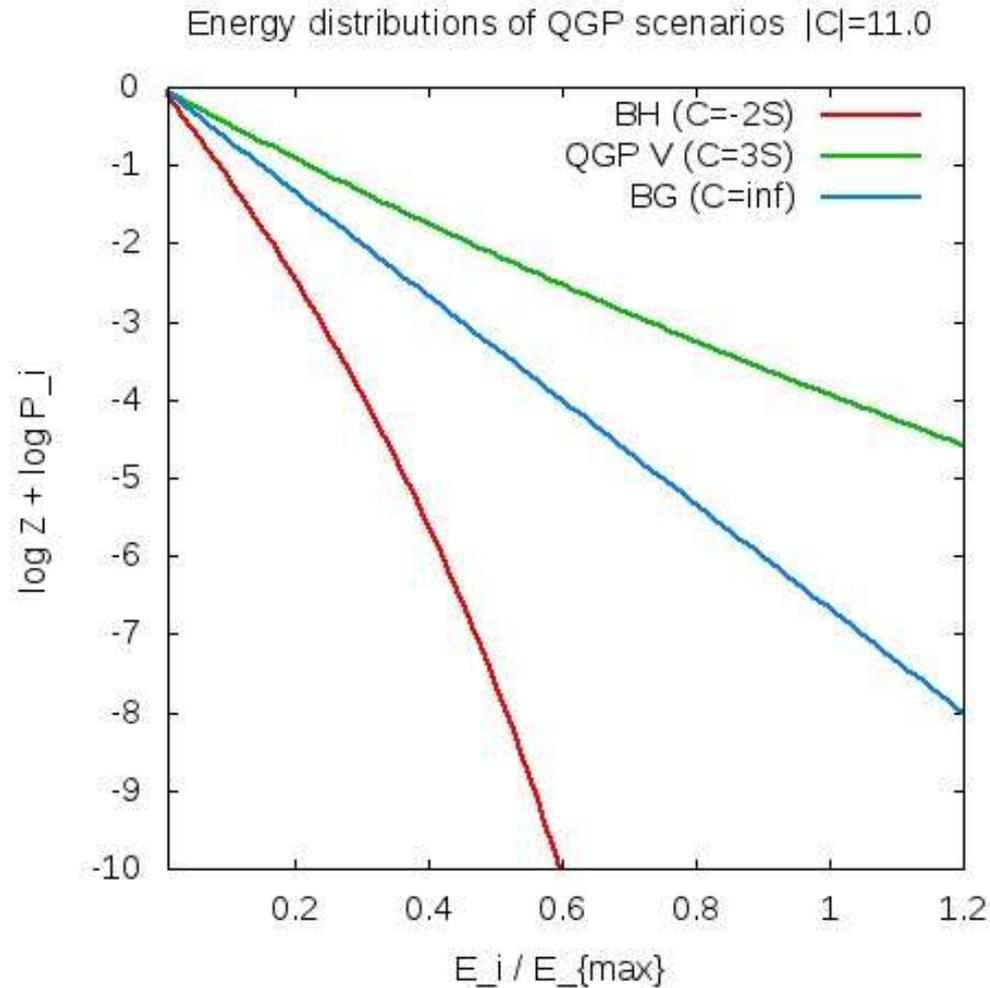
$$T_{fit} = T$$

- Schwarzschild black hole:

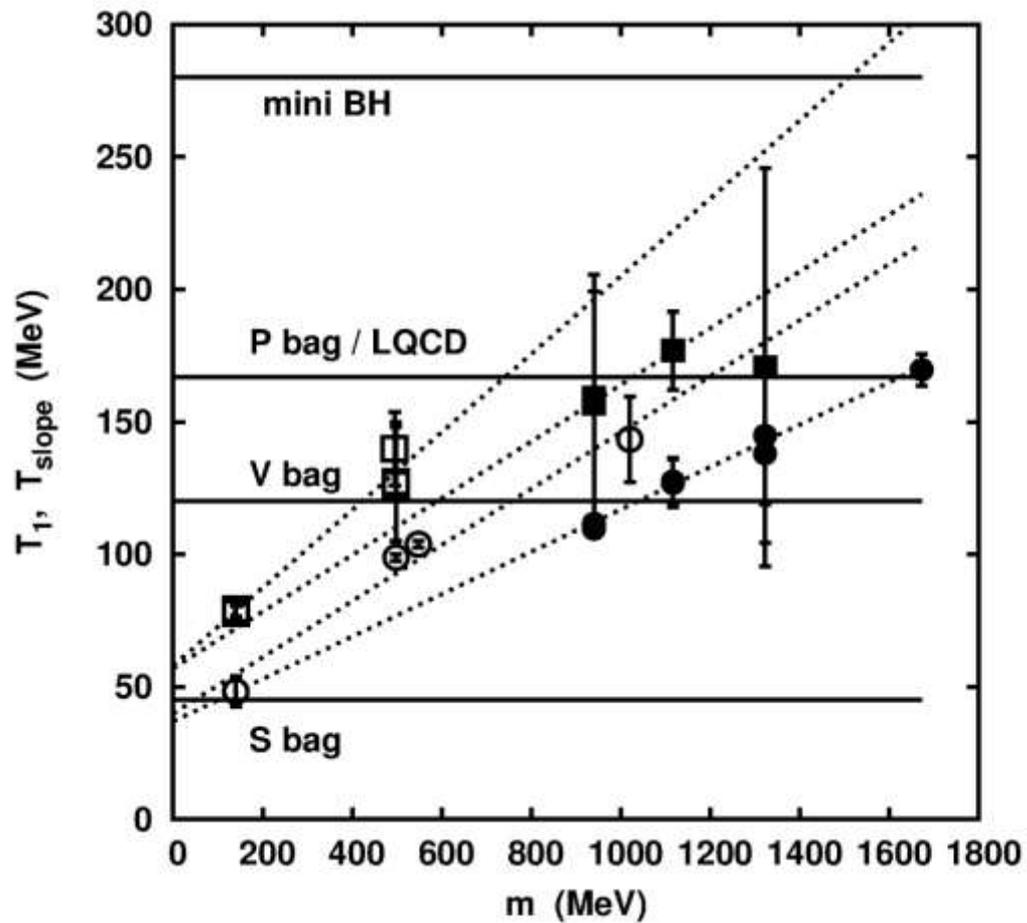
$$\text{➤ } S = \alpha E^2, \quad \frac{1}{T} = 2\alpha E, \quad C = -2\alpha E^2 = -2S$$

$$T_{fit} = T \lim_{C \rightarrow \infty} e^{-S/C} = e^{1/2} \approx 1.65$$

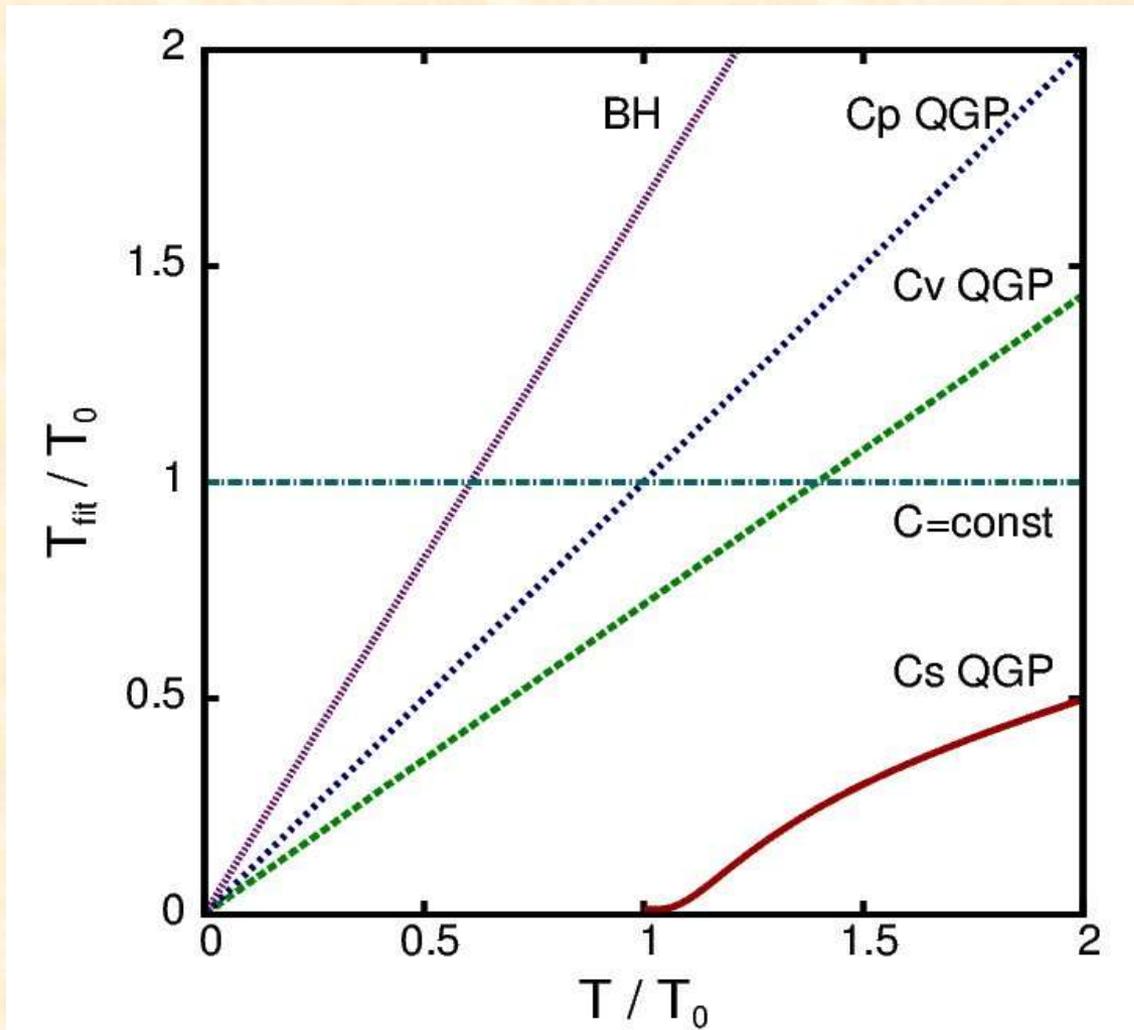
# Black hole or a QGP bag?



# Fitted slopes



# Reservoir models



# Conclusion

- ***Improved Canonical Approach***: assumes statistical entanglement by using optimal  $L(S)$  agreeing to higher order with microcanonical
- **Universally treats *finite heat capacity reservoirs*** (but includes the infinite ones)
- **Tsallis entropy is  $L(S)$ , Rényi entropy is  $S$**
- **Fitted Boltzmann-Gibbs temperature may differ from that of the reservoir**
- **For QGP  $T = 175$  MeV  $V=\text{const}$  fit  $T = 125$  MeV,  $S=\text{const}$  QGP smaller, Yang-Mills fit  $T$  same, mini BH fit  $T = 288$  MeV**

# LECTURE SIX ABOUT (ADVANCED) STATISTICAL PHYSICS

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**Thermalization**

**Pre-thermalization**

**Pseudo-thermalization**

**Our view**

## Unruh gamma radiation at RHIC?

Arxiv: 1111.4817  
Phys.Lett. B  
708:276, 2012

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(Dated: October 23, 2011)

Varying the proposition that acceleration itself would simulate a thermal environment, we investigate the semiclassical coherent photon radiation as a possible telemetric thermometer of accelerated charges. Based on the classical Jackson formula we obtain the equivalent photon intensity spectrum stemming from a constantly accelerated charge and demonstrate its resemblances to a thermal distribution for high transverse momenta. The transverse slope temperature *differs* from the famous Unruh formula: it is larger by a factor of  $\pi$ . We compare the resulting direct photon spectrum with experimental data for AuAu collisions at RHIC and speculate about further, analytically solvable acceleration histories.

PACS numbers: 24.10.Pa, 25.75.Ag, 25.20.Lj

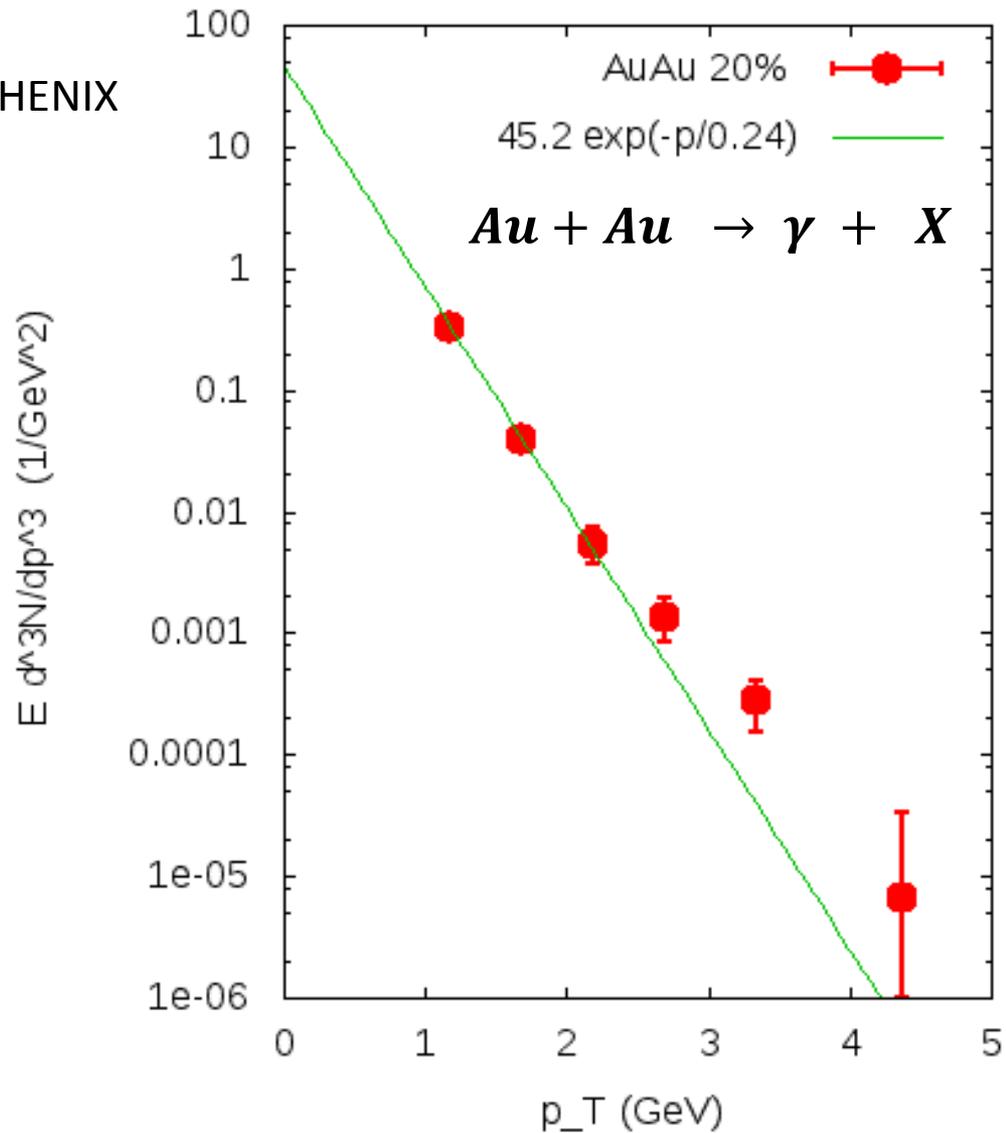
Keywords: Thermal models, Unruh temperature, bremsstrahlung, photon spectra

# Why Photons (gammas) ?

- **Zero mass:** flow – Doppler, easy kinematics
- **Color neutral:** escapes strong interaction
- **Couples to charge:**  $Z / A$  sensitive
- **Classical** field theory also predicts spectra

# Experimental motivation: apparently thermal photons

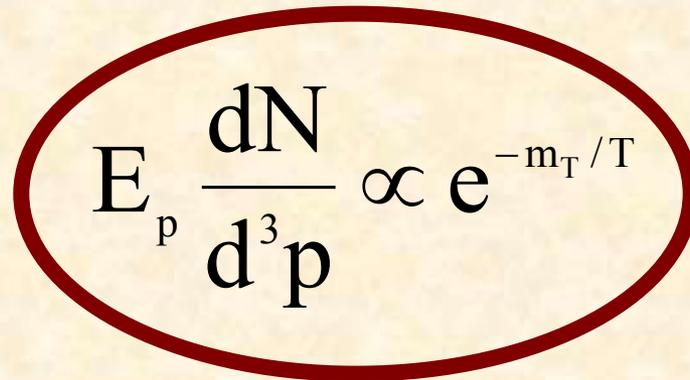
RHIC: PHENIX



# Theoretical motivation

- **Deceleration** due to stopping
- Schwinger formula + Newton + **Unruh** = Boltzmann

$$E_p \frac{dN}{d^3p} \propto e^{-2\pi m_T^2 / qE}, \quad qE = m_T a, \quad T = \frac{a}{2\pi}$$


$$E_p \frac{dN}{d^3p} \propto e^{-m_T/T}$$

Satz, Kharzeev, ...

# Soft bremsstrahlung

- Jackson formula for the amplitude:

$$\vec{A} = K \int e^{i\phi} \frac{d}{dt} \left( \frac{\vec{n} \times (\vec{n} \times \vec{\beta})}{1 - \vec{n} \cdot \vec{\beta}} \right) dt$$

$\vec{u}$

With  $K^2 = \frac{e^2}{8\pi c^2}$ ,  $\vec{\beta} = \frac{\vec{v}}{c} = \frac{1}{c} \frac{d\vec{r}}{dt}$

and the retarded phase  $\phi = \omega \left( t - \frac{\vec{n} \cdot \vec{r}}{c} \right) = \mathbf{k} \cdot \mathbf{x}$

# Soft bremsstrahlung

- Covariant notation:

$$k = (\omega, \omega \vec{n}) = k_{\perp} (\cosh \eta, \sinh \eta, \cos \psi, \sin \psi)$$

$$u = (\gamma, \gamma \vec{v}) = (\cosh \xi, \sinh \xi, 0, 0)$$

$$\mathfrak{N} = \int e^{i\varphi} \frac{d}{d\tau} \left( \frac{\epsilon \cdot u}{k \cdot u} \right) d\tau$$


*IR div, coherent effects*

*Feynman graphs*

**The Unruh effect cannot be calculated by any finite number of Feynman graphs!**

# Kinematics, source trajectory

- Rapidity:  $\beta = \frac{v}{c} = \tanh(\xi + \xi_0)$   
 $\xi = \frac{g}{c} \tau$

Trajectory:

$$t = t_0 + \frac{c}{g} (\sinh(\xi + \xi_0) - \sinh \xi_0)$$

$$z = z_0 + \frac{c^2}{g} (\cosh(\xi + \xi_0) - \cosh \xi_0)$$

**Let us denote  $\xi + \xi_0$  by  $\xi$  in the followings!**

# Kinematics, photon rapidity

- Angle and rapidity:

$$\cos \theta = \tanh \eta$$

$$\sin \theta = \frac{1}{\cosh \eta}$$

$$\cot \theta = \sinh \eta$$

$$\eta = \ln \cot \frac{\theta}{2}$$

# Kinematics, photon rapidity

- Doppler factor:

$$\mathbf{k} \cdot \mathbf{u} = \omega \gamma (1 - \beta \cos \theta) = \omega \frac{\cosh(\xi - \eta)}{\cosh \eta} = \frac{d\phi}{d\tau}$$

Phase:

$$\phi = \frac{\omega c}{g} \frac{\sinh(\xi - \eta)}{\cosh \eta} = \ell k_{\perp} \sinh(\xi - \eta)$$

Magnitude of projected velocity:

$$u = \frac{\sinh \xi}{\cosh(\xi - \eta)}, \quad \frac{du}{d\xi} = \frac{\cosh \eta}{\cosh^2(\xi - \eta)}$$

# Intensity, photon number

Amplitude as an integral over rapidities on the trajectory:

$$\vec{A} = K \vec{e} \int_{\xi_1}^{\xi_2} e^{i\ell k_{\perp} \sinh(\xi - \eta)} \frac{\cosh \eta}{\cosh^2(\xi - \eta)} d\xi$$

Here  $\ell = \frac{c^2}{g}$  is a characteristic length.

# Intensity, photon number

Amplitude as an integral over infinite rapidities on the trajectory (velocity goes from  $-c$  to  $+c$ ):

$$\vec{A} = 2K \vec{e} \ell k_{\perp} \cosh \eta K_1(\ell k_{\perp})$$

With  $K_1$  Bessel function!

$$\frac{dN}{k_{\perp} dk_{\perp} d\eta d\psi} = \frac{4\alpha_{EM}}{\pi} \ell^2 K_1^2(\ell k_{\perp})$$

**Flat in rapidity !**

# Photon spectrum, limits

Amplitude as an integral over infinite rapidities on the trajectory (velocity goes from  $-c$  to  $+c$ ):

$$\frac{dN}{k_{\perp} dk_{\perp} d\eta d\psi} = \frac{4\alpha_{EM}}{\pi} \frac{1}{k_{\perp}^2} \quad \text{for } \ell k_{\perp} \rightarrow 0$$

$$\frac{dN}{k_{\perp} dk_{\perp} d\eta d\psi} = 2\alpha_{EM} \frac{\ell}{k_{\perp}} e^{-2\ell k_{\perp}} \quad \text{for } \ell k_{\perp} \rightarrow \infty$$

# Apparent temperature

- High -  $k_{\perp}$  infinite proper time acceleration:

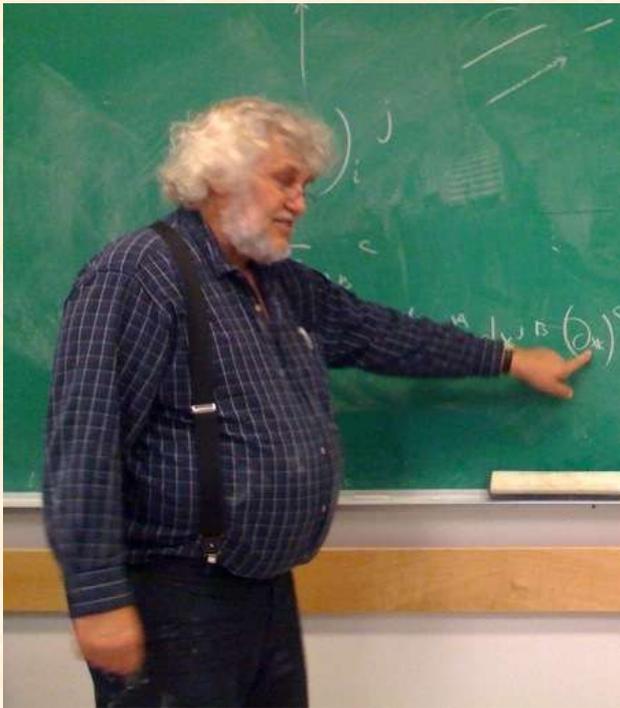
$$k_B T = \frac{\hbar c}{2\ell} = \frac{\hbar g}{2c} = \pi k_B T^{Unruh}$$

Connection to Unruh:

$$\frac{du}{d\tau} \rightarrow e^{-i\nu\tau}$$

proper time Fourier analysis of a  
monochromatic wave

# Unruh temperature



Unruh

- Entirely classical effect
- Special Relativity suffices

$$I(\nu) \propto \left| \int e^{i \left[ \int \omega \sqrt{\frac{1-V(\tau)/c}{1+V(\tau)/c}} d\tau - \nu\tau \right]} d\tau \right|^2$$



$$I(\nu) \propto \left| \int_0^\infty e^{i c \omega z / g} z^{-i \nu c / g - 1} dz \right|^2 \propto \frac{1}{e^{2\pi \nu / g} - 1}$$

# Unruh temperature

Planck-interpretation:

$$\frac{2\pi c}{g} \nu = \frac{\hbar \nu}{k_B T}$$

The temperature in  
Planck units:

$$T = \frac{g}{2\pi}$$

The temperature  
more commonly:

$$k_B T = \frac{\hbar}{c} \frac{g}{2\pi} = M_P g \cdot \frac{L_P}{2\pi}$$

# Unruh temperature

Small for Newtonian gravity

$$g = \frac{GM}{R^2}$$

$$k_B T = \frac{Mc^2}{2\pi} \cdot \frac{L_P^2}{R^2}$$

**On Earth' surface it is  $10^{(-19)}$  eV, while at room temperature about  $10^{(-3)}$  eV.**

# Unruh temperature

Not small in heavy ion collisions

$$g = \frac{c^2}{2L} = \frac{mc^3}{\hbar}$$

$$k_B T = \frac{mc^2}{2\pi}$$

Braking from +c to -c in a Compton wavelength:

$$kT \sim \mathbf{150 \text{ MeV}} \text{ if } mc^2 \sim 940 \text{ MeV (proton)}$$

# Connection to Unruh

$$\frac{dN}{k_{\perp} dk_{\perp} d\eta d\psi} = \frac{\alpha_{EM}}{2\pi k_{\perp}^2 \cosh^2 \eta} \left| \int_{-\infty}^{+\infty} e^{i\phi(\tau)} \frac{du}{d\tau} d\tau \right|^2$$

**Fourier component for the retarded phase:**

$$f_k = \int_{-\infty}^{+\infty} e^{i\phi(\tau)} e^{i\nu\tau} d\tau = \frac{\ell}{c} \int_{-\infty}^{+\infty} e^{i\ell k_{\perp} \sinh \xi} e^{ik\xi} d\xi$$

# Connection to Unruh

**Fourier component for the projected acceleration:**

$$a_k = \int_{-\infty}^{+\infty} \frac{du}{d\tau} e^{i\nu\tau} d\tau = \cosh \eta \int_{-\infty}^{+\infty} \frac{1}{\cosh^2 \xi} e^{ik\xi} d\xi$$

**Photon spectrum in the incoherent approximation:**

$$\frac{dN}{k_{\perp} dk_{\perp} d\eta d\psi} \approx \frac{\alpha_{EM}}{2\pi k_{\perp}^2 \cosh^2 \eta} \int_{-\infty}^{+\infty} |f_k|^2 |a_k|^2 \frac{c}{\ell} \frac{dk}{2\pi}$$

# Connection to Unruh

**Fourier component for the retarded phase at constant acceleration:**

$$f_k = \frac{\ell}{c} \int_{-\infty}^{+\infty} e^{i\ell k_{\perp} \sinh \xi} e^{ik\xi} d\xi = \frac{2\ell}{c} K_{ik}(\ell k_{\perp}) e^{-\pi k/2}$$

**KMS relation and Planck distribution:**

$$f_{-k} = e^{k\pi} f_k^*, \quad |f_{-k}|^2 = e^{2\pi k} |f_k|^2$$

$$-n(-\nu) = e^{2\pi\ell\nu/c} n(\nu) = 1 + n(\nu)$$

$$n(\nu) = \frac{1}{e^{2\pi\ell\nu/c} - 1}$$

# Connection to Unruh

**KMS relation and Planck distribution:**

$$2\pi k = \frac{2\pi c}{g} \nu = \frac{\hbar}{k_B T_U} \nu ;$$

$$T_U = \frac{\hbar}{2\pi k_B c} g$$

# Connection to Unruh

**Note:**

$$a_k = \cosh \eta \frac{k\pi}{\sinh k\pi/2}$$

**It is peaked around  $k = 0$ , but relatively wide! (an unparticle...)**

# Intensity, photon number

Amplitude as an integral over infinite rapidities on the trajectory (velocity goes from  $-c$  to  $+c$ ):

$$\vec{A} = 2K \vec{e} \ell k_{\perp} \cosh \eta K_1(\ell k_{\perp})$$

With  $K_1$  Bessel function!

$$\frac{dN}{k_{\perp} dk_{\perp} d\eta d\psi} = \frac{4\alpha_{EM}}{\pi} \ell^2 K_1^2(\ell k_{\perp})$$

**Flat in rapidity !**

# Transverse flow interpretation

Mathematica knows: ( I derived it using Feynman variables)

$$\int_0^\pi \frac{d\theta}{\sin \theta} K_2 \left( \frac{z}{\sin \theta} \right) = K_1^2 \left( \frac{z}{2} \right) \quad \frac{1}{\sin \theta} = \cosh \eta$$

$$\frac{dN}{k_\perp dk_\perp d\eta d\psi} = \frac{4\alpha_{EM} \hbar c}{\pi(2\pi k_B T_U)^2} \int_{-\infty}^{+\infty} K_2 \left( \frac{\hbar c k_\perp}{\pi k_B T_U} \cosh(\zeta - \eta) \right) d\zeta$$

$$\frac{dN}{k_\perp dk_\perp d\eta d\psi} = \frac{4\alpha_{EM}}{\pi g} \int_{-\infty}^{+\infty} K_2 \left( \frac{k \cdot u_{Bjorken}}{\pi T_{Unruh}} \right) d\zeta$$

Alike Jüttner distributions integrated over the flow rapidity...

# Finite time (rapidity) effects

$$\frac{dN}{k_{\perp} dk_{\perp} d\eta d\psi} = \frac{4K^2}{\hbar k_{\perp}^2} \left| \int_{w_1}^{w_2} \frac{e^{i\ell k_{\perp} w}}{(1+w^2)^{3/2}} \right|^2$$

with  $w = \sinh(\xi - \eta)$

**Short-time deceleration  $\rightarrow$  Non-uniform rapidity distribution;  $\rightarrow$  Landau hydrodynamics**

**Long-time deceleration  $\rightarrow$  uniform rapidity distribution;  $\rightarrow$  Bjorken hydrodynamics**

# Short time constant acceleration

$$\frac{dN}{k_{\perp} dk_{\perp} d\eta d\psi} = \frac{4\alpha_{EM}}{\pi} \frac{1}{k_{\perp}^2} \frac{(w_2 - w_1)^2}{(1 + w_0^2)^3}$$

$$\frac{dN}{k_{\perp} dk_{\perp} d\eta d\psi} = \frac{4\alpha_{EM}}{\pi} \frac{4}{\omega^2} \frac{1}{\cosh^2 \eta}$$

**Non-uniform rapidity distribution;**

**→ Landau hydrodynamics**

# Further analytic results

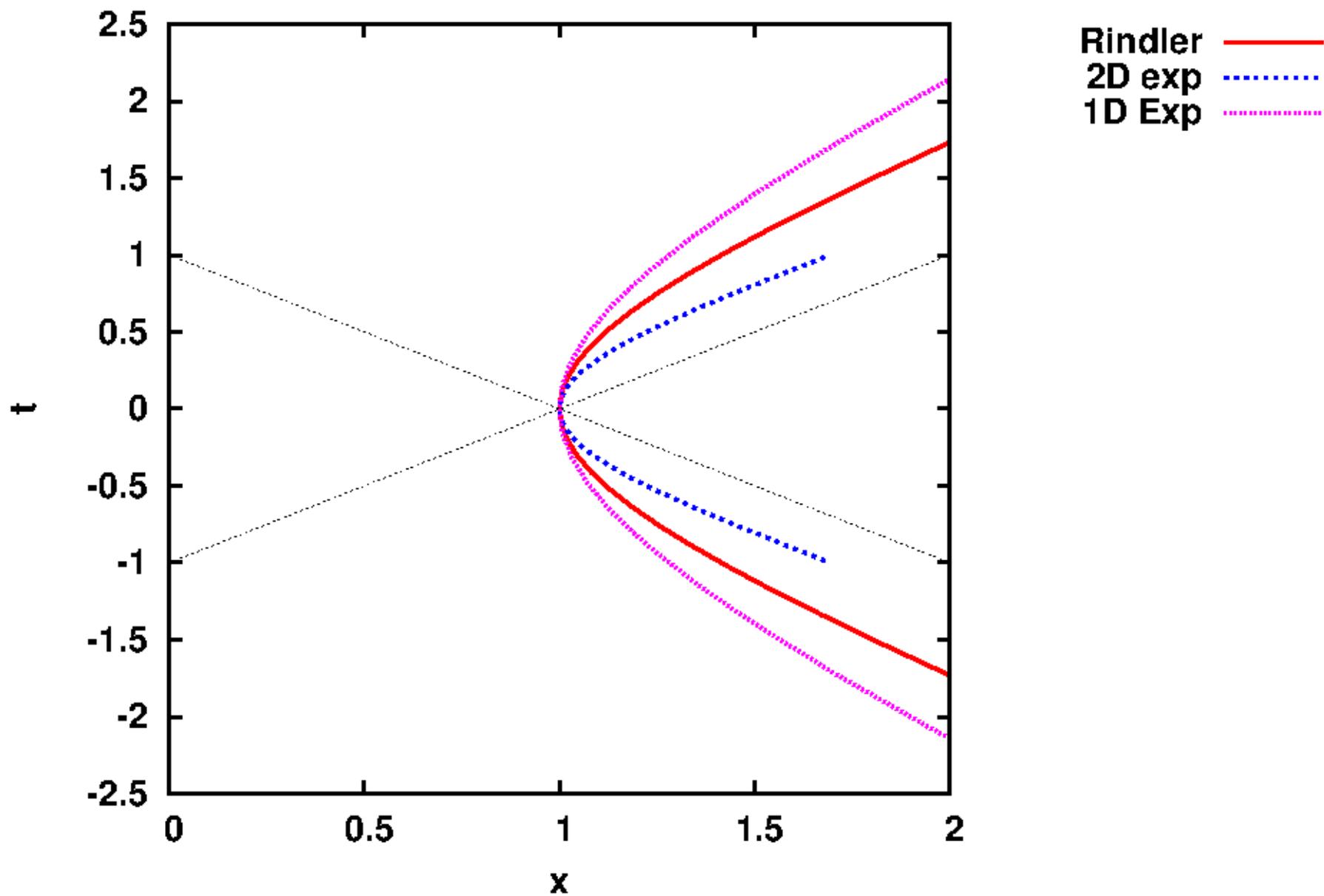
$x(t)$ ,  $v(t)$ ,  $g(t)$ ,  $\tau(t)$ ,  $\mathcal{A}$ ,  $dN/kdkd\eta$  limit

$$\sqrt{1+t^2}, \frac{t}{\sqrt{1+t^2}}, 1, \text{Arc sh } t, bK_1(b), \frac{\ell}{k} e^{-2\ell k}$$

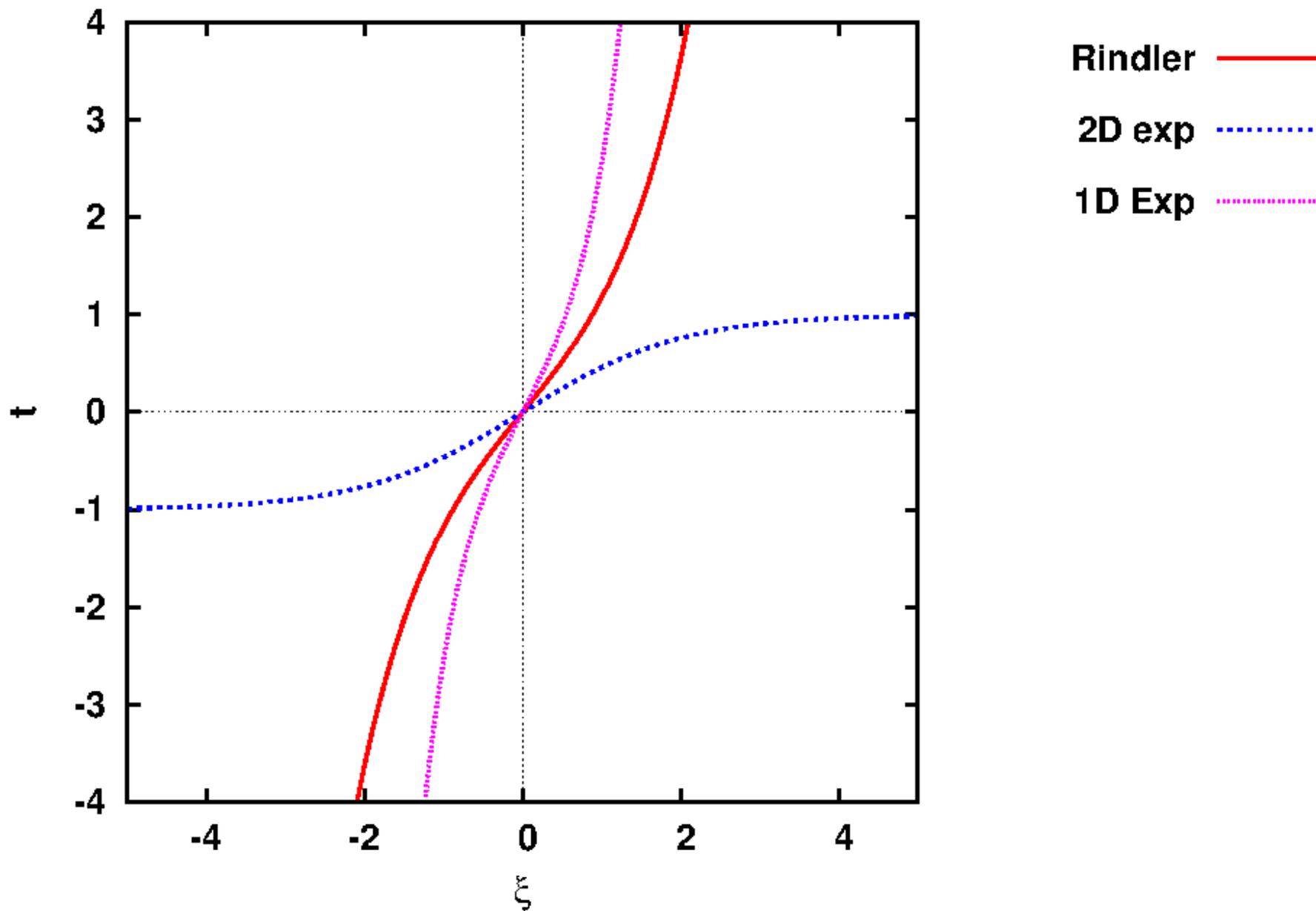
$$1 + \ln(1+t^2), \frac{2t}{1+t^2}, \frac{2(1+t^2)}{(1-t^2)^2}, 2 \text{atn } t - t, be^{-b}, \ell^2 e^{-2\ell k}$$

$$1 + \frac{2t}{\pi} \text{atn} \left( \frac{t}{\pi} \right) - \ln \left( 1 + \frac{t^2}{\pi^2} \right), \frac{2}{\pi} \text{atn} \left( \frac{t}{\pi} \right), \frac{2\gamma^3}{\pi^2 + t^2}, \frac{\pi^2}{2} \int \frac{\sqrt{1-v^2} dv}{\cos^2\left(\frac{\pi v}{2}\right)}, e^{-b}, \frac{1}{k^2} e^{-2\ell k}$$

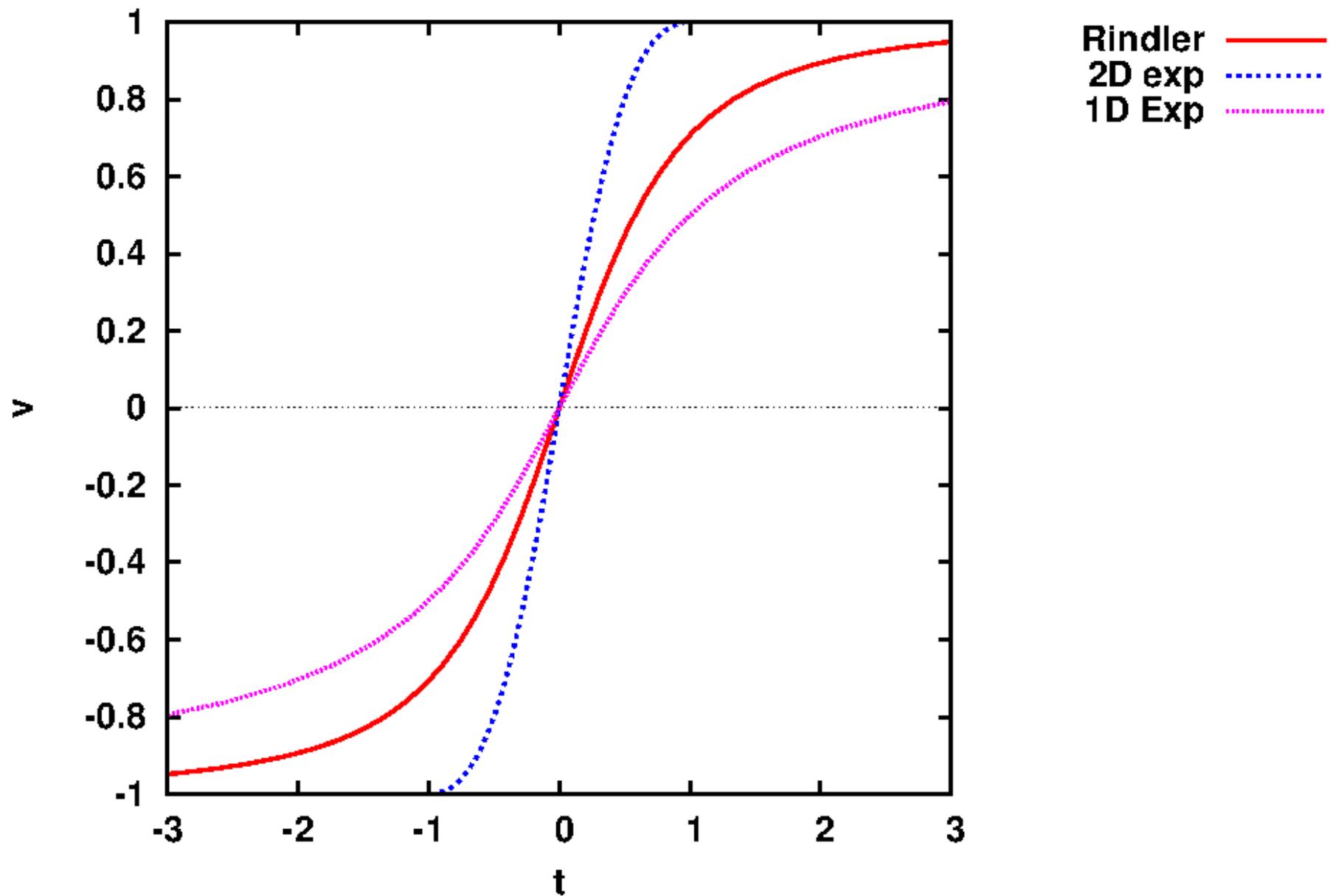
### Spacetime paths for the charge



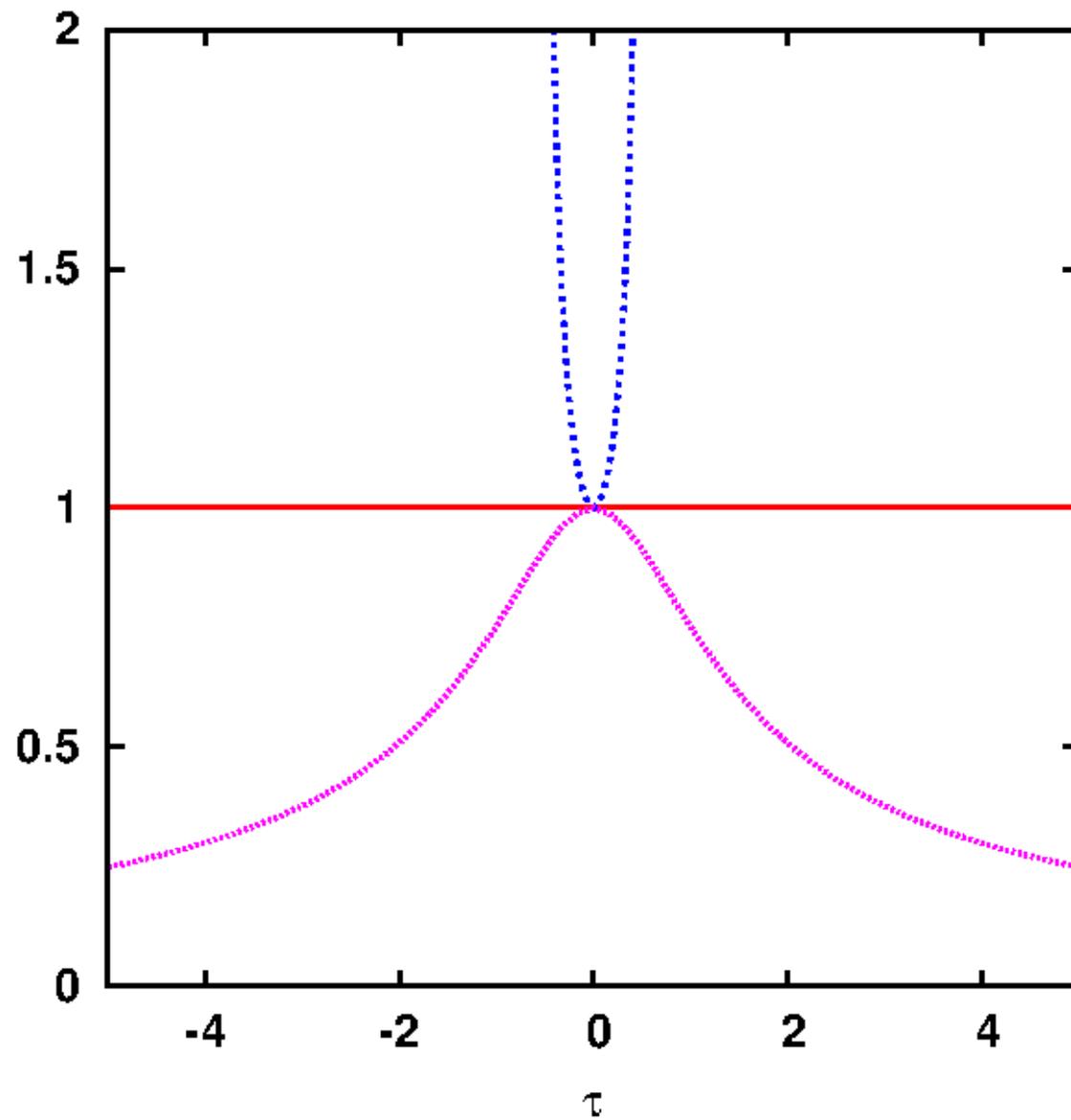
# Time vs Rapidity



Velocity evolution in lab frame

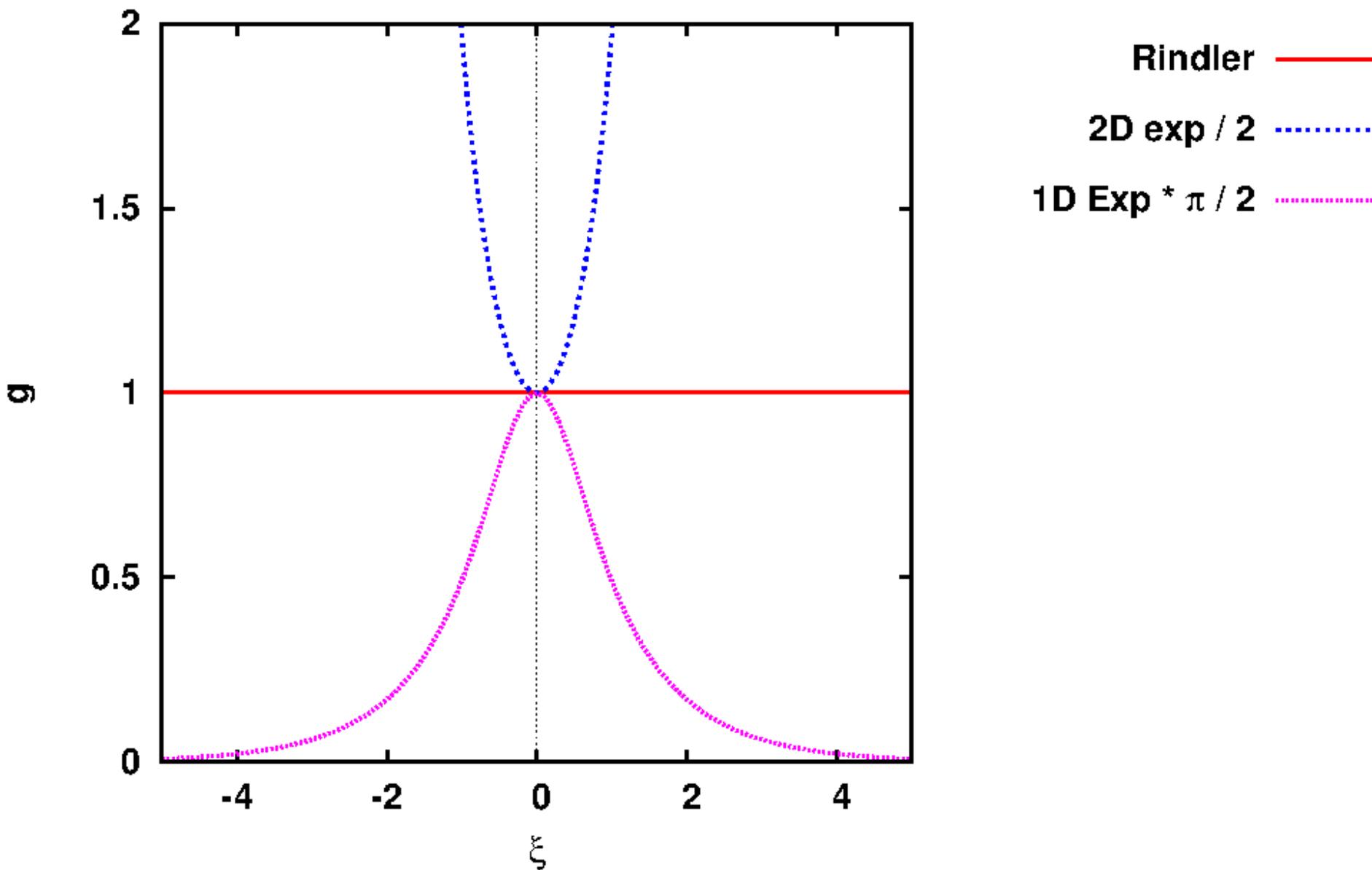


### Acceleration proper

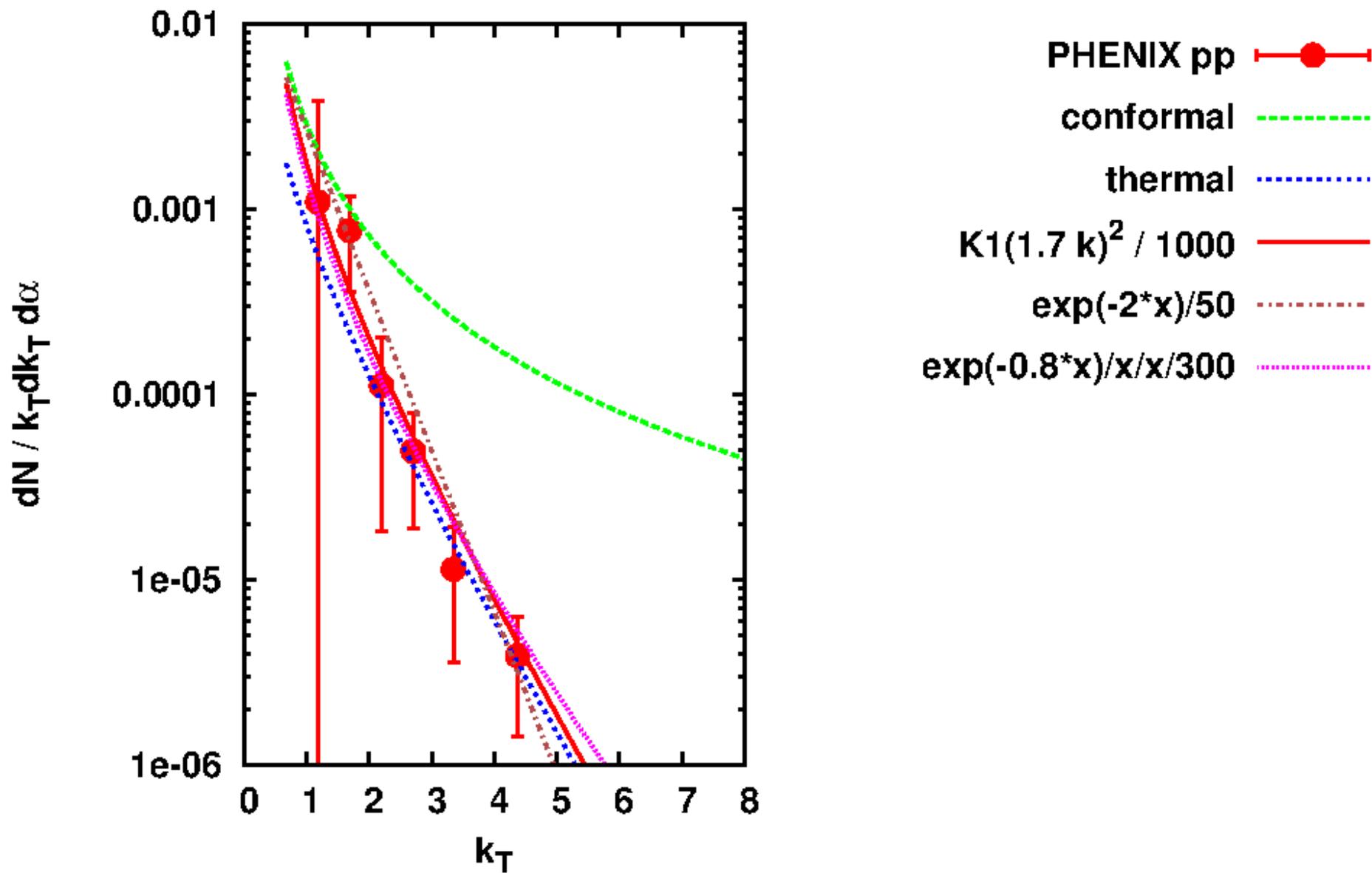


- Rindler ———
- 2D exp / 2 ·····
- 1D Exp \*  $\pi / 2$  ·····

# Acceleration vs Rapidity

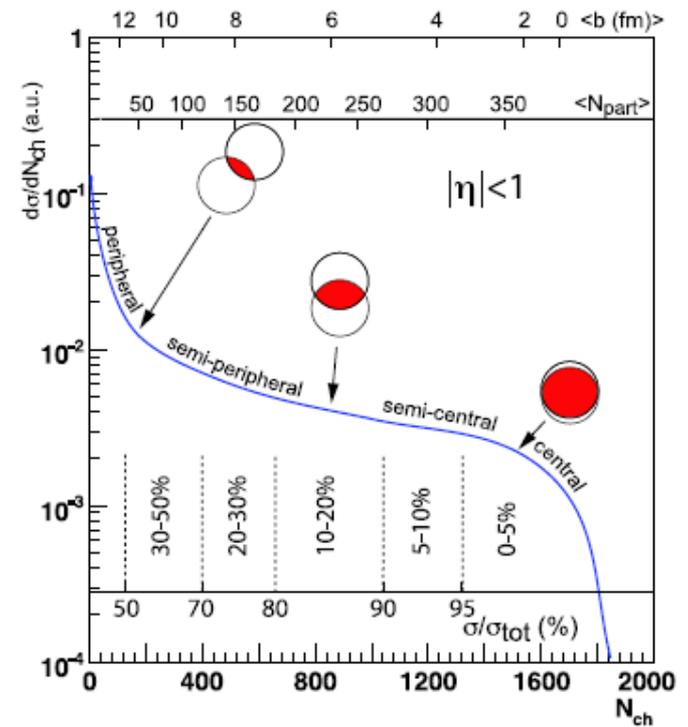
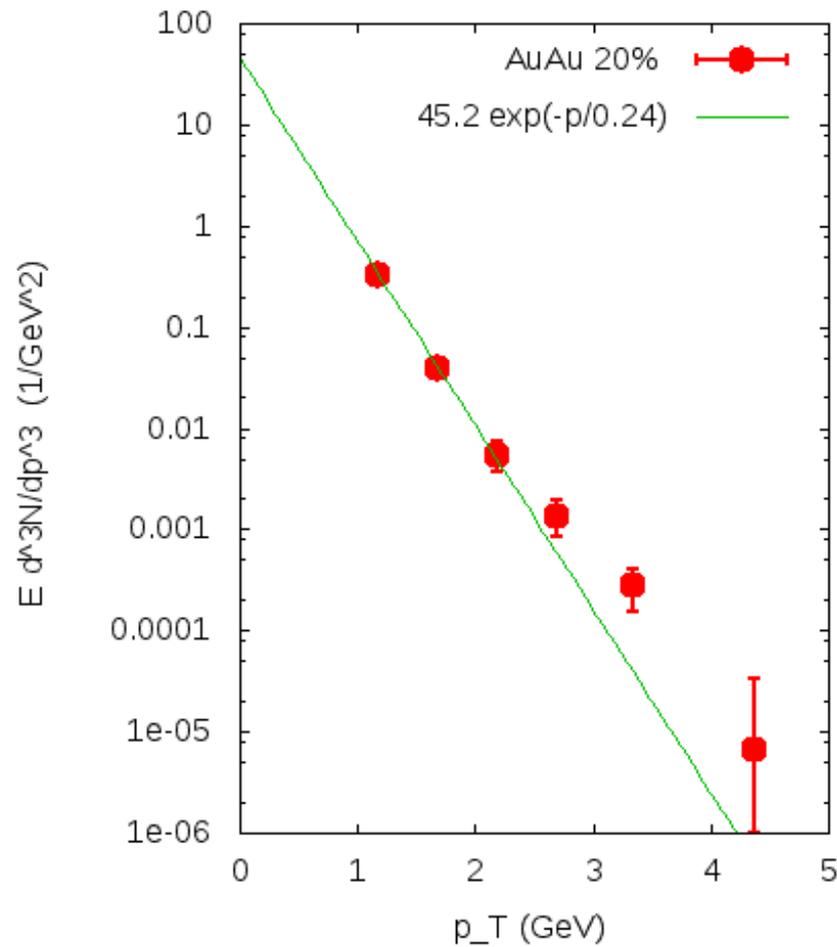


DirectPhoton\_pp3.eps



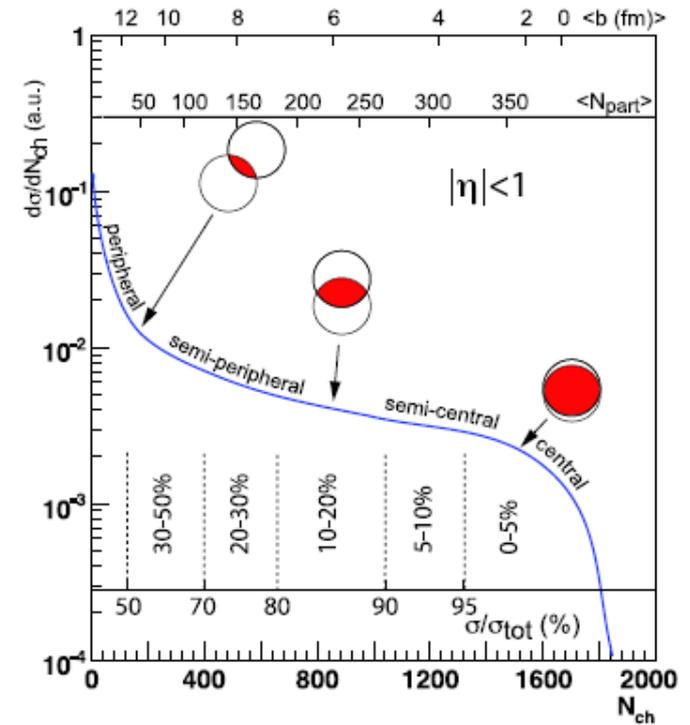
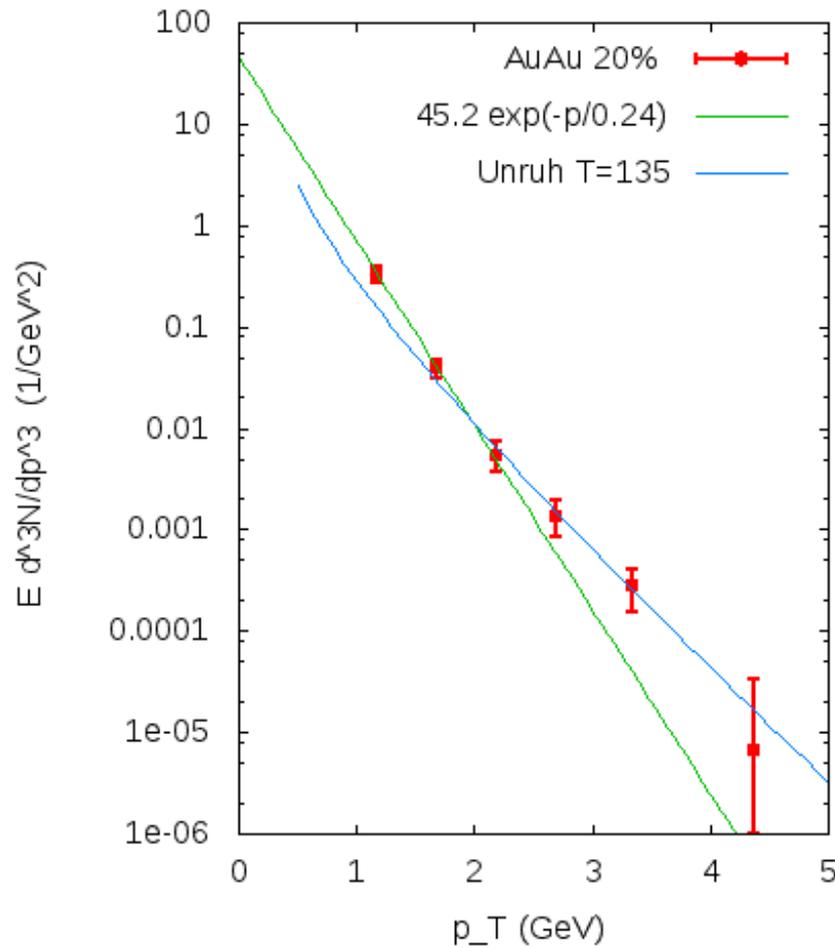
$$Au + Au \rightarrow \gamma + X$$

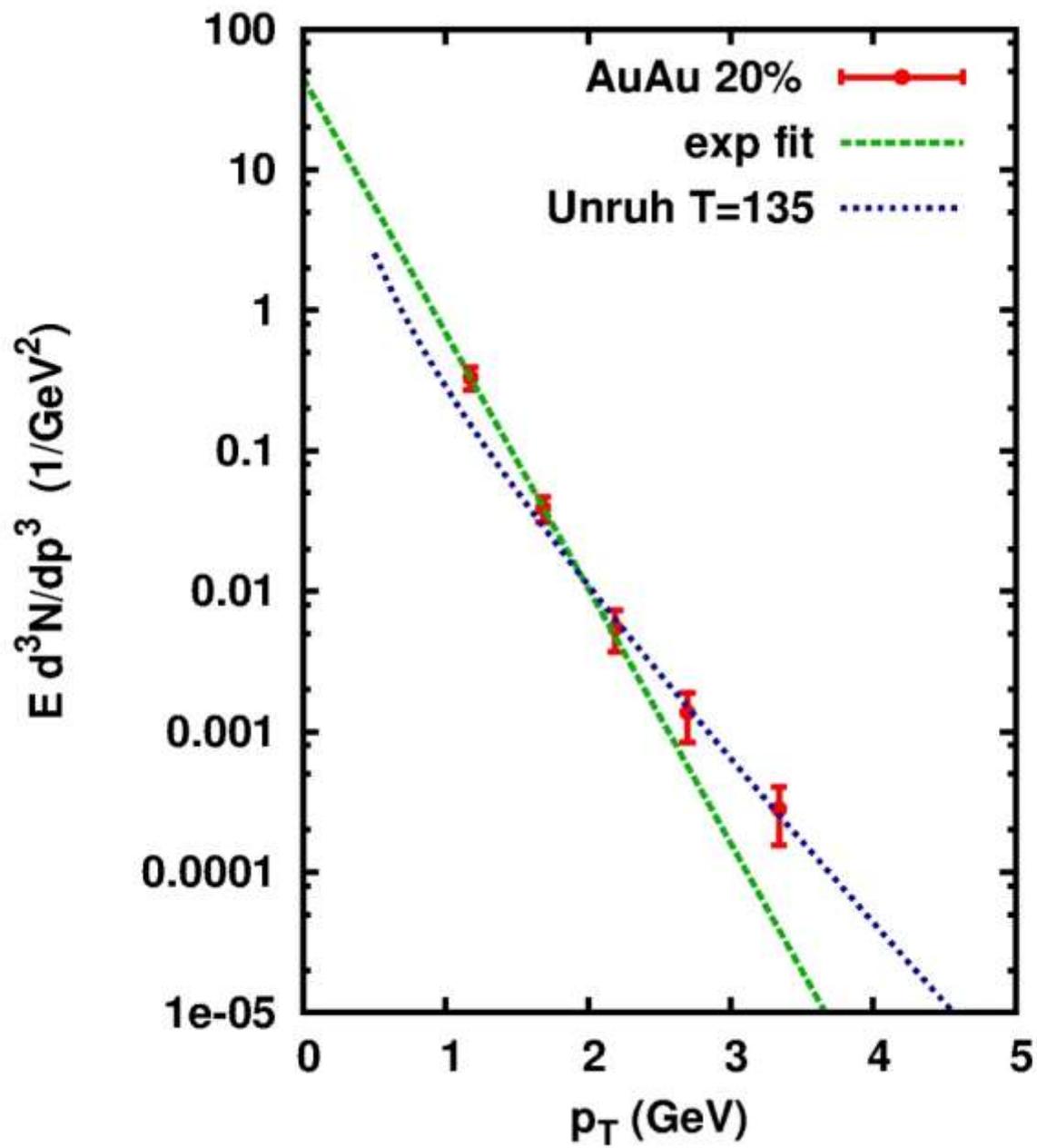
## Glauber model



$$Au + Au \rightarrow \gamma + X$$

## Glauber model





# Summary

- Semiclassical radiation from constant accelerating point charge occurs **rapidity-flat** and **thermal**
- The thermal tail develops at high enough  $k_{\perp}$
- At low  $k_{\perp}$  the **conformal** NLO result emerges
- Finite time/rapidity acceleration leads to **peaked** rapidity distribution, alike **Landau** - hydro
- Exponential fits to surplus over NLO pQCD results reveal a " **$\pi$ -times Unruh-**" temperature

**Discussion**





**Is acceleration a heat container?**